Synopsis of proposed research work entitled

"Study of Some Thermoelastic Problems and its Applications Using Mathematical Software"

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Submitted by

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"Study of Some Thermoelastic Problems and its applications using Mathematical Software"

1. Introduction (Origin of the Research Problem) :

The term **thermoelasticity** deals with the effects of the thermal state of an elastic body on the stress distribution, and also with the effects of the stress on the temperature. Thermoelasticity is the study of the relationship between the elastic properties of a material and its temperature, or between its thermal conductivity and its stresses.

The theory of elasticity was extended to include thermal effects. The theory of thermoelasticity is concerned with the influence of the thermal state of an elastic solid upon the distribution of strain and with the inverse effect, that of deformation upon the thermal state of an elastic medium. Thermoelasticity was stimulated by the various engineering sciences. The theory of thermoelasticity was found in 1837 by **Duhamel** [1] who derived the formulation of boundary value problem and also the derivation of equation for the coupling of the temperature field and the body's deformation. The formulation of thermoelasticity equations is due to **Neumann** [2] in 1985, to **Almansi** [3] in 1897 and **Tedone** [4] in 1906. **Signorini** [5] of 1930 on finite difference deformations should also be mentioned. In 1935, **Biot** [6] analyzed the properties of two-dimensional distributions of thermal stresses and **Goodier** [7] in 1937, introduced the notation of the thermoelastic potential and considered the effect of non-continues temperature fields.

During and after the Second World War, the requirements associated with new technologies contributed to a wave of research on thermal stresses. Gaining knowledge of the distribution of temperature in specific situations, finding the thermal stresses in parts of complex mechanical systems, assessment of allowed stresses in various materials and in various loading conditions, matters of stability, problems of viscoelasticity, of fatigue, and thermal shock, became topics of active research both theoretical and experimental.

2. Review of Literature & Developments in the subject:

Thermoelastic problems are of two types: Direct and Indirect

1. Direct Thermoelastic Problem

The direct thermoelastic problems are consist of temperature, heat transfer conditions prescribed on the surfaces of the body and the conditions at interior points are to be determined. The direct thermoelastic problems are more useful in engineering problem particularly, in the determination of the state of strain in a rectangular plate constituting foundations of containers for hot gases or liquids, in the foundations for furnaces etc.

In 1956, **Biot** [8] developed the coupled theory of thermoelasticity. In this theory, the equations of elasticity and of heat conduction are coupled which deals with the first defect of the uncoupled theory. In 1957, **Nowacki** [9] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. In 1959, **Florence and Goodier** [10] discussed a uniform flow of heat is disturbed by an insulated cavity or an insulated hole in an elastic plate, thermal stresses are produced in the plate. In 1993, **Ashida et al.** [11] discussed a new solution technique for thermoelastic problems of transversely isotropic solids in cylindrical coordinates. In 2002, **Kim and Noda** [12] studied two-dimensional unsteady thermoelastic problems of the FGM infinite hollow cylinder and the deflection of a functionally graded plate under transient thermal loading. In 2009, **Deshmukh et al.** [13] studied the non-homogeneous steady-state heat conduction problem in a thin circular plate and its thermal stresses.

In 2012, **Gaikwad and Ghadle** [14] have studied the non-homogeneous heat conduction problem and its thermal deflection due to internal heat generation in a thin hollow circular disk. In 2013, **Gaikwad** [15] analyzed the thermoelastic deformation of a thin hollow circular disk due to partially distributed heat supply. In 2015, **Gaikwad** [16] studied mathematical modeling and its simulation of a quasi-static thermoelastic problem in a semi-infinite hollow circular disk due to internal heat generation. In 2015, **Gaikwad** [17] studied mathematical modeling of thermoelastic problem in a circular sector disk subject to heat generation. In 2016, **Bhad et al.** [18] studied heat source problem of thermoelasticity in an elliptic plate with thermal bending moments. In 2016, **Bhad et al.** [19] studied heat source thermoelastic problem in a hollow elliptic cylinder under time-reversal principle. In 2016, **Gaikwad** [20] solved two-dimensional steady-state temperature distribution of a thin circular plate due to uniform internal energy generation. In 2017, **Gaikwad** [21] studied steady-state heat conduction problem in a thick circular plate and its thermal stresses.

2. Inverse Thermoelastic Problem

The thermoelasticity is an approach to derive the temperature, displacement and thermal stresses in different types of solid due to heating under various initial and boundary conditions. The inverse thermoelastic problems consist of the determination of the heating medium, the heat flux at the boundary surfaces of the Solids when the conditions of the displacement and stresses were known at the same points of the solid under consideration. The inverse problem was very important in view of its relevance to various industrial machines subjected to heating such as the main shaft of lathe and turbine and the role of rolling mill, turbines subjected to heating and cooling mediums. Also it is useful in measurement of aerodynamic heating.

The investigation of one-dimensional transient thermoelastic problem was done by **Grysa et al.** [22] in 1981, **Grysa and Cialkowski** [23] in 1982, **Kozlowski and Grysa** [24] in 1982. The heating temperature and the heat flux on the surface of an isotropic infinite slab were derived by them. In 1989, **Noda et al.** [25] discussed an analytical method for an inverse problem of three-dimensional transient thermoelasticity in a transversely isotropic solid by applying the Laplace and Fourier transforms with newly designed potential function and the practically applicability of the methods in engineering problems of great importance was illustrated.

In 2002, **Ashida et al.** [26] discussed the inverse transient thermoelastic problem for composite circular disc. In 2004, **Ashida et al.** [27] solved an inverse problem of transient thermoelasticity in a piezoceramic hollow cylinder of crystal class. In 2005, **Gaikwad** [28] solved the inverse problem of thermoelasticity in a thin isotropic circular plate by determining the unknown temperature gradient, temperature distribution and the thermal deflection on the edge of the circular plate. In 2011, **Gaikwad and Ghadle** [29] solved an inverse problem for the quasi-static thermoelastic system in a thin clamped circular plate. In 2011, **Gaikwad and Ghadle** [30] solved an inverse heat conduction problem in a thick annular disc.

3. Objectives of Research /Proposed Hypothesis:

1. To study in details direct and inverse thermoelastic problems.
2. To study existing methods.

3. To formulate the new boundary value problems of heat conduction problems on various solid and study their thermoelasticity.
4. To construct new methods for solving thermoelastic problems and obtain alternative solutions of the problems.
5. To study applications related with engineering field.

4. Methodology to be adopted:

The first few months will be devoted for survey and collection of relevant research papers, review articles, books, journals etc. Some latest and relevant books may be purchased/collected for up gradation of knowledge. Considering the ample computer resource one has available today, it is reasonable to use general purpose Maple, Mathematica, Mathcad. The main emphasis, in the present study, will be on analytical and numerical approach for investigating problems arising in thermoelastictly. The work, which will be carried out during the proposed study, will be published in the reputed Journals.

5. Importance of Study/Society Application:

Interdisciplinary Relevance:

The issue has great importance because it is an interdisciplinary work related with the branch of science and technology like Physics, Mechanical Engineering, and Civil Engineering etc. The isotropic and anisotropic thermoelastic problems to be considered have important industrial applications.

Significance of the Study:

The direct thermoelastic problems have applications in the field of Aeronautics. The high velocity of modern aircrafts gives rise to aeronautic heating. In turn, this produces intense thermal stresses by lowering the elastic limit and reduces the strength of aircraft structure and spacecrafts. While the inverse thermoelastic problem is very important in view of its relevance to various industrial machines subjected to heating such as main shaft of lathe, turbine, and roll of rolling mill. It is also useful to analyze the experimental data and measurement of aerodynamic heating. Thus thermoelastic problems play pivotal role in the development of science and technology especially

aerospace engineering. The study of such problems will tremendously enhance the speed of the development in the industry, which will culminate in the overall development of the country.

Recently **Kalpana Chawla** and her team died due to the overheating and loss of insulation in the space shuttle. Such type of problems can be sorted out by studying the direct and inverse thermoelastic problems.

6. Proposed Work Plan/Formulation & Structure of Study:

Year-Wise plan of work and targets to be achieved:

First year

i) To collect the references in relevant field.

ii) To study the references in relevant field.

iii) To attend the seminars/Conferences/Workshops on relevant topic.

Second year

i) To attend the seminars/Conferences/Workshops on relevant topic.

ii) To formulate new thermoelastic problem on different solids.

iii) To study existing and new technique will be applied on these problems.

iv) To construct the mathematical models for various solids and study their thermoelasticity.

v) To check whether these models practically feasible or not, will be discussed with engineering researchers.

Third year

- To obtain the applications of thermoelastic problems on different solids and analyzed their thermoelasticity.
- To publish the work in reputed journals.
- To submit the work to the authority.

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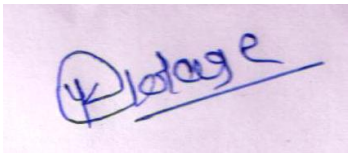

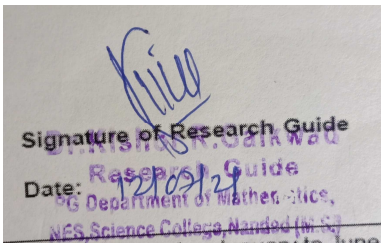

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